AQA A-Level Physics: Periodic Motion – Calculation Questions

Praneel Physics

1. A mass of 0.5 kg oscillates with amplitude 0.2 m and period 1.5 s. Calculate its maximum speed. **(P)**

Working and Answer:

$v_{max} = \frac{2\pi A}{T} = \frac{2\pi \times 0.2}{1.5} \approx 0.84\, m/s.$

2. A simple pendulum has length 1.0 m. Calculate its period (g = 9.81 m/s²). **(P)**

Working and Answer:

$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.0}{9.81}} \approx 2.01\,s.$

3. Calculate the spring constant for a mass of 0.2 kg that oscillates with period 0.8 s. **(P)**

Working and Answer:

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 \times 0.2}{0.8^2} \approx 12.3\,N/m.$

4. A mass-spring system has k = 50 N/m and m = 2 kg. Calculate its angular frequency. **(P)**

Working and Answer:

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{2}} = 5\, rad/s.$

5. Calculate the frequency of oscillation for a simple pendulum with length 0.25 m (g = 9.81 m/s^2). **(P)**

Working and Answer:

$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.25}} \approx 0.997\, Hz.$

6. A mass of 0.1 kg oscillates with amplitude 0.05 m and angular frequency 10 rad/s. Calculate its maximum acceleration. **(P)**

Working and Answer:

$a_{max} = \omega^2 A = 10^2 \times 0.05 = 5\, m/s^2$.

7. A mass-spring system has m = 0.5 kg and completes 20 oscillations in 25 s. Calculate the spring constant. **(PP)**

Working and Answer:

1. Period $T = \frac{25}{20} = 1.25\, s$
2. $k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 \times 0.5}{1.25^2} \approx 12.6\, N/m$.

8. A pendulum on Earth has period 2.0 s. Calculate its period on Mars where g = 3.71 m/s^2. **(PP)**

Working and Answer:

1. Earth: $2 = 2\pi\sqrt{\frac{L}{9.81}} \Rightarrow L \approx 0.993\,m$
2. Mars: $T = 2\pi\sqrt{\frac{0.993}{3.71}} \approx 3.25\,s.$

9. An oscillating mass has displacement x = 0.1cos(4πt). Calculate its speed at t = 0.5 s. **(PP)**

Working and Answer:

1. $\omega = 4\pi\,rad/s$
2. $v = -\omega A\sin(\omega t) = -4\pi \times 0.1\sin(4\pi \times 0.5) = 0\,m/s.$

10. A spring stretches 0.1 m when a 0.5 kg mass is hung from it. Calculate its oscillation period. **(PP)**

Working and Answer:

1. $k = \frac{mg}{x} = \frac{0.5 \times 9.81}{0.1} = 49.05\, N/m$
2. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.5}{49.05}} \approx 0.634\, s.$

11. A simple pendulum has period 1.5 s on Earth. Calculate its length. **(PP)**

Working and Answer:

1. $T = 2\pi\sqrt{\frac{L}{g}}$
2. $L = \frac{gT^2}{4\pi^2} = \frac{9.81 \times 1.5^2}{4\pi^2} \approx 0.559\, m.$

12. Calculate the total energy of a 0.2 kg mass oscillating with amplitude 0.1 m and angular frequency 8 rad/s. **(PP)**

Working and Answer:

1. $E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2} \times 0.2 \times 8^2 \times 0.1^2 = 0.064\,J.$

13. A mass-spring system has m = 0.4 kg, k = 64 N/m, and amplitude 0.15 m. Calculate: (a) maximum acceleration, (b) speed when displacement is 0.05 m, (c) total energy. **(PPP)**

Working and Answer:

1. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{64}{0.4}} = 12.65\, rad/s$
2. $a_{max} = \omega^2 A = 12.65^2 \times 0.15 \approx 24\, m/s^2$
3. $v = \omega\sqrt{A^2 - x^2} = 12.65\sqrt{0.15^2 - 0.05^2} \approx 1.79\, m/s$
4. $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 64 \times 0.15^2 = 0.72\, J.$

14. A pendulum clock runs slow by 1 minute per day. Calculate the required length adjustment (original length = 0.993 m). **(PPP)**

Working and Answer:

1. Actual period $T_{actual} = \frac{86400}{86400-60} \times 2.0 \approx 2.0023\, s$
2. $L_{new} = L_{old} \times \left(\frac{T_{new}}{T_{old}}\right)^2 = 0.993 \times \left(\frac{2.0023}{2.0}\right)^2 \approx 0.995\, m$
3. Need to shorten by $0.995 - 0.993 = 0.002\, m$.

15. A 0.5 kg mass oscillates with x = 0.2sin(5t). Calculate: (a) maximum kinetic energy, (b) potential energy when x = 0.1 m. **(PPP)**

Working and Answer:

1. $\omega = 5\,rad/s,\ A = 0.2\,m$
2. $KE_{max} = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2} \times 0.5 \times 5^2 \times 0.2^2 = 0.25\,J$
3. $PE = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2} \times 0.5 \times 5^2 \times 0.1^2 = 0.0625\,J.$

16. A spring-mass system has total energy 0.5 J, mass 0.1 kg, and amplitude 0.2 m. Calculate: (a) angular frequency, (b) maximum speed, (c) period. **(PPP)**

Working and Answer:

1. $E = \frac{1}{2}m\omega^2 A^2 \Rightarrow \omega = \sqrt{\frac{2E}{mA^2}} = \sqrt{\frac{1}{0.1 \times 0.04}} \approx 15.81\, rad/s$
2. $v_{max} = \omega A \approx 3.16\, m/s$
3. $T = \frac{2\pi}{\omega} \approx 0.397\, s.$

17. A simple pendulum has length 1.0 m on Earth. Calculate: (a) period on Earth, (b) length needed for same period on Moon (g = 1.62 m/s²). **(PPP)**

Working and Answer:

1. Earth: $T = 2\pi\sqrt{\frac{1.0}{9.81}} \approx 2.01\,s$
2. Moon: $L = \frac{gT^2}{4\pi^2} = \frac{1.62\times 2.01^2}{4\pi^2} \approx 0.165\,m.$

18. A mass-spring system has m = 0.25 kg, k = 40 N/m, and is displaced 0.1 m. Calculate: (a) period, (b) maximum speed, (c) speed at x = 0.05 m. **(PPP)**

Working and Answer:

1. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{0.25}} = 12.65\, rad/s$
2. $T = \frac{2\pi}{\omega} \approx 0.497\, s$
3. $v_{max} = \omega A = 12.65 \times 0.1 = 1.265\, m/s$
4. $v = \omega\sqrt{A^2 - x^2} = 12.65\sqrt{0.1^2 - 0.05^2} \approx 1.095\, m/s.$

19. A damped oscillator has initial amplitude 0.2 m and decreases to 0.1 m in 10 s. Calculate: (a) damping constant, (b) percentage energy lost per oscillation if period is 2 s. **(PPPP)**

Working and Answer:

1. $A = A_0 e^{-bt/2m} \Rightarrow 0.1 = 0.2e^{-b\times 10/2m}$
2. $\ln(0.5) = -5b/m \Rightarrow b/m = 0.1386\, s^{-1}$
3. Energy ratio: $\left(\frac{A_1}{A_0}\right)^2 = e^{-bT/m} = e^{-0.1386\times 2} \approx 0.757$
4. Energy lost per period: $1 - 0.757 = 24.3\%$.

20. A forced oscillator has natural frequency 2 Hz, driven at 2.5 Hz with damping constant 0.4 s^{-1}. Calculate: (a) quality factor Q, (b) amplitude ratio at resonance vs driving frequency. **(PPPP)**

Working and Answer:

1. $\omega_0 = 2\pi \times 2 = 4\pi\, rad/s$
2. $Q = \frac{\omega_0}{b} = \frac{4\pi}{0.4} \approx 31.4$
3. Amplitude ratio $\approx Q = 31.4$ (for small damping).

21. A pendulum in a lift has period 2.0 s at rest. Calculate its period when the lift: (a) accelerates up at 2 m/s^2, (b) accelerates down at 2 m/s^2. **(PPPP)**

Working and Answer:

1. Effective g up: $g' = 9.81 + 2 = 11.81\, m/s^2$, $T = 2\pi\sqrt{\frac{L}{11.81}} \approx 1.83\, s$
2. Effective g down: $g' = 9.81 - 2 = 7.81\, m/s^2$, $T = 2\pi\sqrt{\frac{L}{7.81}} \approx 2.24\, s$
3. Original length: $L = \frac{gT^2}{4\pi^2} = \frac{9.81\times4}{4\pi^2} \approx 0.993\, m$.

22. A mass-spring system (m = 0.5 kg, k = 200 N/m) is damped with b = 2 kg/s. Calculate: (a) damping ratio, (b) period of damped oscillation, (c) time for amplitude to halve. **(PPPP)**

Working and Answer:

1. Critical damping: $b_c = 2\sqrt{km} = 2\sqrt{200 \times 0.5} \approx 20\,kg/s$
2. Damping ratio: $\zeta = \frac{b}{b_c} = 0.1$ (underdamped)
3. $\omega' = \omega_0\sqrt{1-\zeta^2} = \sqrt{\frac{200}{0.5}}\sqrt{1-0.01} \approx 19.9\,rad/s$
4. $T = \frac{2\pi}{\omega'} \approx 0.316\,s$
5. Halving time: $t_{1/2} = \frac{\ln 2 \times 2m}{b} \approx 0.693 \times \frac{1}{2} \approx 0.347\,s.$

23. A physical pendulum is a rod (L = 1 m, m = 0.5 kg) pivoted at one end. Calculate: (a) moment of inertia, (b) period, (c) equivalent simple pendulum length. **(PPPP)**

Working and Answer:

1. $I = \frac{1}{3}mL^2 = \frac{1}{3} \times 0.5 \times 1^2 \approx 0.167\, kg\, m^2$
2. $T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{0.167}{0.5\times 9.81\times 0.5}} \approx 1.64\, s$
3. $L_{eq} = \frac{I}{md} = \frac{0.167}{0.5\times 0.5} \approx 0.668\, m.$

24. A torsional pendulum has wire constant 0.02 Nm/rad and disk moment of inertia 0.001 kg m^2. Calculate: (a) period, (b) angular displacement after $\pi/4$ rad initial twist is released. **(PPPP)**

Working and Answer:

1. $\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{0.02}{0.001}} \approx 4.472\, rad/s$
2. $T = \frac{2\pi}{\omega} \approx 1.405\, s$
3. $\theta(t) = \frac{\pi}{4}\cos(\omega t)$.

25. A damped harmonic oscillator has m = 0.2 kg, k = 80 N/m, b = 1.6 kg/s. Calculate: (a) damping ratio, (b) Q factor, (c) time for energy to drop to 1/e of initial, (d) oscillation frequency. **(PPPPP)**

Working and Answer:

1. $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{80}{0.2}} = 20\,rad/s$
2. Critical damping: $b_c = 2\sqrt{km} = 2\sqrt{80 \times 0.2} = 8\,kg/s$
3. Damping ratio: $\zeta = \frac{b}{b_c} = 0.2$
4. $Q = \frac{1}{2\zeta} = 2.5$
5. Energy decay time: $\tau = \frac{2m}{b} = \frac{0.4}{1.6} = 0.25\,s$
6. $\omega' = \omega_0\sqrt{1-\zeta^2} = 20\sqrt{1-0.04} \approx 19.6\,rad/s.$

26. A physical pendulum consists of a circular hoop (R = 0.3 m, m = 0.4 kg) suspended from a point on its rim. Calculate: (a) moment of inertia, (b) period, (c) center of percussion distance. **(PPPPP)**

Working and Answer:

1. Parallel axis: $I = \frac{3}{2}mR^2 = \frac{3}{2} \times 0.4 \times 0.09 = 0.054\,kg\,m^2$
2. $T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{0.054}{0.4\times9.81\times0.3}} \approx 1.35\,s$
3. $L_{cp} = \frac{I}{md} = \frac{0.054}{0.4\times0.3} = 0.45\,m.$

27. A mass-spring system (m = 0.2 kg, k = 45 N/m) is subject to damping and driving force F = 0.5cos(14t) N. Calculate: (a) resonant frequency, (b) amplitude at resonance if Q = 10, (c) bandwidth. **(PPPPP)**

Working and Answer:

1. $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{45}{0.2}} = 15\, rad/s$
2. $A_{res} = Q \times \frac{F_0}{k} = 10 \times \frac{0.5}{45} \approx 0.111\, m$
3. Bandwidth: $\Delta\omega = \frac{\omega_0}{Q} = 1.5\, rad/s.$

28. A compound pendulum has mass 1 kg, length 1 m, and radius of gyration 0.3 m about its pivot. Calculate: (a) moment of inertia, (b) period, (c) length of equivalent simple pendulum, (d) center of oscillation. **(PPPPP)**

Working and Answer:

1. $I = mk^2 = 1 \times 0.09 = 0.09\,kg\,m^2$
2. $T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{0.09}{1\times9.81\times0.5}} \approx 0.851\,s$
3. $L_{eq} = \frac{I}{mh} = \frac{0.09}{0.5} = 0.18\,m$
4. Center of oscillation: $L_{co} = \frac{k^2}{h} + h = \frac{0.09}{0.5} + 0.5 = 0.68\,m.$